① __Question__: How many 3-letter passwords can you make using 3 of these letters: TCUFROG?

$$\underbrace{7}_{} \cdot \underbrace{6}_{} \cdot \underbrace{5}_{} = 210$$

3 buckets

This is an example of computing

$P(n,r) = {}_nP_r$ = # of permutations (arrangements) of r objects from a set of n objects.

$$P(n,r) = \underbrace{\underline{n} \cdot \underline{(n-1)} \cdot \underline{(n-2)} \cdots \underline{(n-r+1)}}_{r \text{ buckets}}$$

$$P(n,r) = n(n-1)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

eg $$7 \cdot 6 \cdot 5 = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$

$C(n,r) = {}_nC_r = \binom{n}{r}$ = # of combinations (sets) of r objects taken from a set of n objects.

the sets are not ordered.

$$\Rightarrow C(n,r) = \frac{P(n,r)}{\left(\substack{\#\text{ of ways to} \\ \text{rearrange } r \text{ items}}\right)} = \frac{P(n,r)}{P(r,r)} = \frac{P(n,r)}{r!}$$

$$\Rightarrow C(n,r) = \binom{n}{r} = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{r!\,(n-r)!}$$

② If we assembled a 3-person team from our class of 33 student to compete in the Putnam Math Competition, there would be $C(33,3) = \binom{33}{3}$ ways to choose the team. $\frac{33 \cdot 32 \cdot 31}{3 \cdot 2 \cdot 1} = 5456$ ways.

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3 ⓐ How many possible ways can you draw 1 pair when draw two cards from a standard deck of cards?

2 of same rank.

52 cards in a deck
4 suits – hearts, clubs, spades, diamonds
each suit has 13 cards – 2, 3, ..., 10, Jack, Queen, King, Ace. ranks
4 in each rank.

$\underbrace{52,\ 3,}$

counting order ⇒ must divide by $2! = 2$

$\Rightarrow 26 \cdot 3 = \boxed{78 \text{ ways}}$

3b What is the probability of drawing a pair when you draw two cards from a standard deck?

Ans: $\dfrac{\#\text{ways of getting one pair}}{\#\text{ways of getting any two cards}}$

$\Rightarrow \text{Prob} = \frac{78}{\binom{52}{2}} = \frac{78}{\left(\frac{52 \cdot 51}{2 \cdot 1}\right)} = \frac{78}{1326}$

$= .05882\ldots$

Another way of getting (3a).

$$\underbrace{13}_{\substack{\#\text{ of}\\ \text{different}\\ \text{ranks}}} \cdot \underbrace{\binom{4}{2}}_{\substack{\text{choose 2}\\ \text{from}\\ \text{rank.}}} = 13 \cdot 6 = \boxed{78}.$$

$C(4,2)$

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(4) What are the number of ways to get a full house when drawing 5 cards from a standard deck?
(A Full house = 3 of one rank, 2 of a different rank.)

$\underbrace{\binom{13}{1}}_{\text{rank}}$ _ _ _ _ _

$$\binom{13}{1} \cdot \binom{4}{3} \cdot \binom{12}{1}\binom{4}{2} = 13 \cdot 4 \cdot 12 \cdot 6 = \boxed{3{,}744}$$

- $\binom{13}{1}$: choice of 3-card rank.
- $\binom{4}{3}$: (choose 3 cards)
- $\binom{12}{1}$: choice of 2-card rank.
- $\binom{4}{2}$: (choose 2 cards)

$$\frac{4 \cdot 3}{2 \cdot 1} = \frac{12}{2} = 6$$

$$\frac{13!}{1!\,12!} = 13$$

Good formulas

We discovered:

$$\boxed{\begin{aligned} \binom{n}{n-1} &= \binom{n}{1} = n \\ \binom{n}{k} &= \binom{n}{n-k} = \frac{n!}{k!\,(n-k)!} \end{aligned}}$$

(5) How ways can you get a straight when choosing 5 cards from a standard deck?

↑ 5 #s in a row

example:

$$(3\heartsuit, 4\diamondsuit, 5\spadesuit, 6\heartsuit, 7\clubsuit)$$

$$\underbrace{36}_{\text{lowest rank.}} \cdot \underbrace{4} \cdot \underbrace{4} \cdot \underbrace{4} \cdot \underbrace{4}$$ ← any straight, including straight flush (straight plus all in the same suit.)

$$\Rightarrow \cancel{\frac{36 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{5!}}$$ ← something is wrong here!

given any straight, we can put them in order in our hand.

ans $36 \cdot 4^4$ different ways.

(Subtract $36 \cdot 1^4$ for straight flush) $\Rightarrow 36(4^4 - 1) = 9180$

(6) How many different ways can we choose a committee of 4 from our class with 2 juniors and 2 sophomores?

(Our class has 5 soph's, and 15 juniors). and the whole class has 33 students.

$$\underline{\text{soph}}: \binom{5}{2} = \frac{5 \cdot 4}{2 \cdot 1} = 10$$

$$\underline{\text{juniors}}: \binom{15}{2} = \frac{15 \cdot 14}{2 \cdot 1} = 105$$

$$\text{Total \# of ways} = 10 \cdot 105 = \boxed{1050}.$$

7. What are # of possible soccer formations (D − M − F) if there are at least 1 ~~of~~ of each type out of 10 players?